$\int \frac{1}{x^4} dx$	$\int 2x\sqrt{1+x^2} dx$	$\int \frac{1+tan^2 x}{tanx} dx$
$\int \sqrt[4]{x^3} dx$	$\int - sinx \cos^2 xdx$	$\int (tan\ x + tan^3 x)dx$
$\int \frac{\sqrt[3]{x}}{x^2} dx$	$\int \frac{arc\ tan\ x}{1+x^2} dx$	$\int (3x^2-1)\cos(x^3-x)dx$
$\int \frac{3^x}{4^x} dx$	$\int \frac{1}{(x-3)^2} dx$	$\int \frac{2x}{\sqrt{1+x^2}} dx$
$\int \frac{5}{x^2} dx$	$\int \frac{2x+1}{x^2+x-10} dx$	$\int 3x^2 sin\ x^3 dx$
$\int \sqrt{x}dx$	$\int \frac{2sinx \cos x}{1+sin^2 x} dx$	$\int \frac{e^x}{e^x+9} dx$
$\int \frac{1}{2x} dx$	$\int 4x^3 sin(x^4-3)dx$	$\int \frac{1}{\sqrt{1-x^2}\, arc\ sin\ x} dx$
$\int (x^2-3x-1)dx$	$\int \frac{e^{tanx}}{\cos^2 x} dx$	$\int \frac{\cos\sqrt{x}}{2\sqrt{x}} dx$
$\int \frac{\cos x}{7} dx$	$\int \frac{4^{\ln x}}{x} dx$	$\int (2x^3-3x^2+5x-1)dx$
$\int \left(7x^6 + \frac{1}{2\sqrt{x}}\right)dx$	$\int \frac{2x}{1+x^4} dx$	$\int \frac{2x+5}{7x} dx$

$\int (3^{2x} - e^{4x} - 1)dx$	$\int \sqrt{7x-6}\ dx$	$\int \frac{3x^2}{x+7} dx$
$\int \frac{5}{(2x-1)^2} dx$	$\int \frac{3}{\sqrt{5x+8}} dx$	$\int x^3 sin(x^4 - \pi\ )dx$
$\int (2x-3)(2x+3)dx$	$\int \frac{7}{1+4x^2} dx$	$\int 5^{tan\ x}(1+tan^2 x)dx$
$\int \frac{9}{7x+3} dx$	$\int \frac{5}{\sqrt{1-9x^2}} dx$	$\int \frac{x^2}{4\sqrt{1-x^3}} dx$
$\int \frac{2x^3-x^2}{3x^2} dx$	$\int 2x^2\sqrt{1-x^3} dx$	$\int \frac{7}{x^2-8x+16} dx$
$\int \frac{7x}{5x^2-3} dx$	$\int \frac{3}{4+100x^2} dx$	$\int \frac{5\cos\sqrt{x}}{\sqrt{x}} dx$
$\int 5\cos(3x-2)dx$	$\int \cos 5xdx$	$\int \frac{(1+\ln x)^2}{4x} dx$
$\int \frac{1}{5x-12} dx$	$\int \frac{1}{\cos^2 x\ tan\ x} dx$	$\int \frac{1}{x^2} sin\ \frac{1}{x} dx$
$\int \frac{7sin\ \sqrt{x}}{3\sqrt{x}} dx$	$\int sin^4 x \cos x\ dx$	$\int \frac{1}{x^2+4x+5} dx$
$\int \frac{e^{x+1}}{e^x-5} dx$	$\int \frac{2^{x+2}}{2^x-13} dx$	$\int x\sqrt[3]{x^2+1} dx$

$$\int \frac{1}{x^4} dx$$

$$\int \frac{1}{x^4} dx = \int x^{-4} dx = \frac{1}{-3} x^{-3} = -\frac{1}{x^3}$$

$$\int \sqrt[4]{x^3} dx$$

$$\int \sqrt[4]{x^3} dx = \int x^{\frac{3}{4}} dx = \frac{1}{7/4} x^{\frac{7}{4}} = \frac{4\sqrt[4]{x^7}}{7}$$

$$\int \frac{\sqrt[3]{x}}{x^2} dx$$

$$\int \frac{\sqrt[3]{x}}{x^2} dx = \int \frac{x^{\frac{1}{3}}}{x^2} dx = \int x^{\frac{1}{3}-2} dx = \int x^{-\frac{5}{3}} dx = \frac{1}{-\frac{2}{3}} x^{-\frac{2}{3}} = -\frac{3}{2\sqrt[3]{x}}$$

$$\int \frac{3^x}{4^x} dx$$

$$\int \frac{3^x}{4^x} dx = \int \left(\frac{3}{4}\right)^x dx = \frac{\left(\frac{3}{4}\right)^x}{\ln \frac{3}{4}}$$

$$\int \frac{5}{x^2} dx$$

$$\int \frac{5}{x^2} dx = 5\int x^{-2} dx = 5\frac{1}{-1} x^{-1} = -\frac{5}{x}$$

$$\int \sqrt{x} dx$$

$$\int \sqrt{x} dx = \int x^{\frac{1}{2}} dx = \frac{1}{\frac{3}{2}} x^{\frac{3}{2}} = \frac{2\sqrt{x^3}}{3}$$

$$\int \frac{1}{2x} dx$$

$$\int \frac{1}{2x} dx = \frac{1}{2}\int \frac{1}{x} dx = \frac{1}{2}\ln x$$

$$\int \left(x^2 - 3x - 1\right) dx$$

$$\int \left(x^2 - 3x - 1\right) dx = \frac{x^3}{3} - 3\frac{x^2}{2} - x$$

$$\int \frac{\cos x}{7} dx$$

$$\int \frac{\cos x}{7} dx = \frac{1}{7}\int \cos x dx = \frac{1}{7} sinx$$

$$\int \left(7x^6 + \frac{1}{2\sqrt{x}}\right) dx$$

$$\int\left(7x^6+\frac{1}{2\sqrt{x}}\right)dx=7\frac{1}{7}x^7+\frac{1}{2}\frac{1}{\frac{1}{2}}x^{\frac{1}{2}}=x^7+\sqrt{x}$$

$$\int 2x\sqrt{1+x^2}\,dx$$

$$\int 2x\sqrt{1+x^2}\,dx=\begin{bmatrix}t=1+x^2\\ dt=2xdx\end{bmatrix}=\int\sqrt{t}dt=\frac{1}{\frac{3}{2}}t^{\frac{3}{2}}=\frac{2\sqrt[3]{t^2}}{3}=\frac{2\sqrt[3]{1+x^2}}{3}$$

$$\int -sinx\cos^2 xdx$$

$$\int -sinx\cos^2 xdx=\begin{bmatrix}t=\cos x\\ dt=-sinxdx\end{bmatrix}=\int t^2dt=\frac{t^3}{3}=\frac{\cos^3 x}{3}$$

$$\int\frac{arc\ \ tan\ \ x}{1\ +\ x^{\ 2}}dx$$

$$\int\frac{arc\ tan\ x}{1+x^2}dx=\begin{bmatrix}t=arc\ tan\ x\\ dt=\frac{1}{1+x^2}dx\end{bmatrix}=\int tdx=\frac{t^2}{2}=\frac{arc\ tan^2 x}{2}$$

$$\int\frac{1}{(x-3)^2}dx$$

$$\int\frac{1}{(x-3)^2}dx=\begin{bmatrix}t=x-3\\ dt=dx\end{bmatrix}=\int\frac{1}{t^2}dt=\frac{1}{-1}t^{-1}=-\frac{1}{x-3}$$

$$\int\frac{2x+1}{x^2+x-10}dx$$

$$\int\frac{2x+1}{x^2+x-10}dx=\begin{bmatrix}t=x^2+x-10\\ dt=(2x+1)dx\end{bmatrix}=\int\frac{1}{t}dt=\ln t=\ln\left(x^2+x-10\right)$$

$$\int\frac{2sinx\cos x}{1+sin^2x}dx$$

$$\int\frac{2sinx\cos x}{1+sin^2x}dx=\begin{bmatrix}t=1+sin^2x\\ dt=2sinx\cos xdx\end{bmatrix}=\int\frac{1}{t}dt=\ln t=\ln\left(1+sin^2x\right)$$

$$\int 4x^3sin\left(x^4-3\right)dx$$

$$\int 4x^3sin\left(x^4-3\right)dx=\begin{bmatrix}t=x^4-3\\ dt=4x^3dx\end{bmatrix}=\int sin\ t\ dt=-\cos t=-\cos\left(x^4-3\right)$$

$$\int\frac{e^{tanx}}{\cos^2 x}dx$$

$$\int\frac{e^{tanx}}{\cos^2 x}dx=\begin{bmatrix}t=tanx\\ dt=\frac{1}{\cos^2 x}dx\end{bmatrix}=\int e^t dt=e^t=e^{tanx}$$

$$\int\frac{4^{\ln x}}{x}dx$$

$$\int\frac{4^{\ln x}}{x}dx=\begin{bmatrix}t=\ln x\\ dt=\frac{1}{x}dx\end{bmatrix}=\int 4^t dt=\frac{4^t}{\ln 4}=\frac{4^{\ln x}}{\ln 4}$$

$$\int \frac{2x}{1+x^4}dx$$

$$\int \frac{2x}{1+x^4}dx = \begin{bmatrix} t = x^2 \\ dt = 2xdx \end{bmatrix} = \int \frac{1}{1+t^2}dt = arc\ tan\ t = arc\ tan\ x^2$$

$$\int \frac{1+tan^2 x}{tanx}dx$$

$$\int \frac{1+tan^2 x}{tanx}dx = \begin{bmatrix} t = tanx \\ dt = (1+tan^2 x)dx \end{bmatrix} = \int \frac{1}{t}dt = \ln t = \ln(tan\ x)$$

$$\int (tan\ \ x + tan^3 x)dx$$

$$\int (tan\ x + tan^3 x)dx = \int tan\ x(1+tan^2 x)dx = \begin{bmatrix} t = tanx \\ dt = (1+tan^2 x)dx \end{bmatrix} = \int tdt = \frac{t^2}{2} = \frac{tan^2 x}{2}$$

$$\int (3x^2-1)\cos(x^3-x)dx$$

$$\int (3x^2-1)\cos(x^3-x)dx = \begin{bmatrix} t = x^3 - x \\ dt = (3x^2-1)dx \end{bmatrix} = \int \cos tdt = sin\ t = sin(x^3-x)$$

$$\int \frac{2x}{\sqrt{1+x^2}}dx$$

$$\int \frac{2x}{\sqrt{1+x^2}}dx = \begin{bmatrix} t = 1+x^2 \\ dt = 2xdx \end{bmatrix} = \int \frac{dt}{\sqrt{t}} = \int t^{-\frac{1}{2}}dt = \frac{1}{1/2}t^{1/2} = 2\sqrt{t} = 2\sqrt{1+x^2}$$

$$\int 3x^2 sin\ x^3 dx$$

$$\int 3x^2 sin\ x^3 dx = \begin{bmatrix} t = x^3 \\ dt = 3x^2 dx \end{bmatrix} = \int sin\ t\ dt = -\cos t = -\cos x^3$$

$$\int \frac{e^x}{e^x+9}dx$$

$$\int \frac{e^x}{e^x+9}dx = \begin{bmatrix} t = e^x + 9 \\ dt = e^x dx \end{bmatrix} = \int \frac{dt}{t} = \ln t = \ln(e^x+9)$$

$$\int \frac{1}{\sqrt{1-x^2}\,arc\ sin\ x}dx$$

$$\int \frac{1}{\sqrt{1-x^2}\,arc\ sin\ x}dx = \begin{bmatrix} t = arc\ sin\ x \\ dt = \frac{1}{\sqrt{1-x^2}}dx \end{bmatrix} = \int \frac{dt}{t} = \ln t = \ln(arc\ sin\ x)$$

$$\int \frac{\cos\sqrt{x}}{2\sqrt{x}}dx$$

$$\int \frac{\cos\sqrt{x}}{2\sqrt{x}}dx = \begin{bmatrix} t = \sqrt{x} \\ dt = \frac{1}{2\sqrt{x}}dx \end{bmatrix} = \int \cos t\ dt = sin\ t = sin\ \sqrt{x}$$

$$\int (2x^3 - 3x^2 + 5x - 1)dx$$

$$\int (2x^3 - 3x^2 + 5x - 1)dx = \frac{1}{2}x^4 - x^3 + \frac{5}{2}x^2 - x$$

$$\int \frac{2x+5}{7x}dx$$

$$\int \frac{2x+5}{7x}dx = \int \left(\frac{2}{7} + \frac{5}{7}x^{-1} \right) dx = \frac{2}{7}x + \frac{5}{7}\ln x$$

$$\int \left(3^{2x} - e^{4x} - 1\right) dx$$

$$\int \left(3^{2x} - e^{4x} - 1\right) dx = \begin{bmatrix} t = 2x & z = 4x \\ dt = 2dx & dz = 4dx \end{bmatrix} = \int 3^t \frac{dt}{2} - \int e^z \frac{dz}{4} - \int dx = \frac{3^{2x}}{2\ln 3} - \frac{e^{4x}}{4} - x$$

$$\int \frac{5}{(2x-1)^2}dx$$

$$\int \frac{5}{(2x-1)^2}dx = \begin{bmatrix} t = 2x-1 \\ dt = 2dx \end{bmatrix} = \int \frac{5}{t^2}\frac{dt}{2} = \frac{5}{2}\frac{1}{-1}t^{-1} = -\frac{5}{2(2x-1)}$$

$$\int (2x-3)(2x+3)dx$$

$$\int (2x-3)(2x+3)dx = \int \left(4x^2 - 9\right)dx = \frac{4}{3}x^3 - 9x$$

$$\int \frac{9}{7x+3}dx$$

$$\int \frac{9}{7x+3}dx = \begin{bmatrix} t = 7x+3 \\ dt = 7dx \end{bmatrix} = \int \frac{9}{t}\frac{dt}{7} = \frac{9}{7}\ln t = \frac{9}{7}\ln(7x+3)$$

$$\int \frac{2x^3 - x^2}{3x^2}dx$$

$$\int \frac{2x^3 - x^2}{3x^2}dx = \int \left(\frac{2}{3}x - \frac{1}{3} \right) dx = \frac{2}{3}\frac{1}{2}x^2 - \frac{1}{3}x = \frac{x^2 - x}{3}$$

$$\int \frac{7x}{5x^2 - 3}dx$$

$$\int \frac{7x}{5x^2 - 3}dx = \begin{bmatrix} t = 5x^2 - 3 \\ dt = 10xdx \end{bmatrix} = \frac{7}{10}\int \frac{dt}{t} = \frac{7}{10}\ln t = \frac{7}{10}\ln(5x^2 - 3)$$

$$\int 5\cos(3x-2)dx$$

$$\int 5\cos(3x-2)dx = \begin{bmatrix} t = 3x-2 \\ dt = 3dx \end{bmatrix} = \int 5\cos t \frac{dt}{3} = \frac{5}{3} sin\ t = \frac{5}{3} sin(3x-2)$$

$$\int \frac{1}{5x-12}dx$$

$$\int \frac{1}{5x-12}dx = \begin{bmatrix} t = 5x-12 \\ dt = 5dx \end{bmatrix} = \int \frac{1}{t}\frac{dt}{5} = \frac{1}{5}\ln t = \frac{1}{5}\ln(5x-12)$$

$$\int \frac{7sin\ \sqrt{x}}{3\sqrt{x}}dx$$

$$\int \frac{7sin\ \sqrt{x}}{3\sqrt{x}}dx = \begin{bmatrix} t = \sqrt{x} \\ dt = \dfrac{dx}{2\sqrt{x}} \end{bmatrix} = \int \frac{7sin\ t}{3}2dt = \frac{14}{3}\int sin\ t\ dt = -\frac{14}{3}\cos t = -\frac{14}{3}\cos\sqrt{x}$$

$$\int \frac{e^{x+1}}{e^x - 5}dx$$

$$\int \frac{e^{x+1}}{e^x-5}dx = e\int \frac{e^x}{e^x-5}dx = \begin{bmatrix} t = e^x - 5 \\ dt = e^x dx \end{bmatrix} = e\int \frac{dt}{t} = e\ln t = e\ln\left(e^x-5\right)$$

$$\int \sqrt{7x-6}\ dx$$

$$\int \sqrt{7x-6}\ dx = \begin{bmatrix} t = 7x-6 \\ dt = 7dx \end{bmatrix} = \int \frac{1}{7}\sqrt{t}\ dt = \frac{1}{7}\frac{1}{3/2}t^{3/2} = \frac{2}{21}\sqrt{7x-6}$$

$$\int \frac{3}{\sqrt{5x+8}}dx$$

$$\int \frac{3}{\sqrt{5x+8}}dx = \begin{bmatrix} t = 5x+8 \\ dt = 5dx \end{bmatrix} = 3\int \frac{1}{5}t^{-1/2}dt = \frac{3}{5}\cdot\frac{1}{1/2}t^{1/2} = \frac{6}{5}\sqrt{t} = \frac{6}{5}\sqrt{5x+8}$$

$$\int \frac{7}{1+4x^2}dx$$

$$\int \frac{7}{1+4x^2}dx = 7\int \frac{1}{1+(2x)^2}dx = \begin{bmatrix} t = 2x \\ dt = 2dx \end{bmatrix} = 7\int \frac{1}{1+t^2}\frac{dt}{2} = \frac{7}{2}arc\ tan\ t = \frac{7}{2}arc\ tan\ (2x)$$

$$\int \frac{5}{\sqrt{1-9x^2}}dx$$

$$\int \frac{5}{\sqrt{1-9x^2}}dx = \begin{bmatrix} t = 3x \\ dt = 3dx \end{bmatrix} = 5\int \frac{1}{\sqrt{1-t^2}}\frac{dt}{3} = \frac{5}{3}arc\ sin\ t = \frac{5}{3}arc\ sin\ (3x)$$

$$\int 2x^2\sqrt{1-x^3}dx$$

$$\int 2x^2\sqrt{1-x^3}dx = \begin{bmatrix} t = 1-x^3 \\ dt = -3x^2dx \end{bmatrix} = \int -\frac{2}{3}\sqrt{t}\ dt = -\frac{2}{3}\frac{1}{3/2}t^{3/2} = -\frac{4}{9}\sqrt{\left(1-x^3\right)^3}$$

$$\int \frac{3}{4+100x^2}dx$$

$$\int \frac{3}{4+100x^2}dx = 3\int \frac{1}{4+100x^2}dx = \frac{3}{4}\int \frac{1}{1+25x^2} = \begin{bmatrix} t = 5x \\ dt = 5dx \end{bmatrix} = \frac{3}{4}\int \frac{1}{5}\frac{1}{1+t^2}dt = \frac{3}{20}arc\ tan\ (5x)$$

$$\int \cos 5x dx$$

$$\int \cos 5x dx = \begin{bmatrix} t = 5x \\ dt = 5dx \end{bmatrix} = \int \cos t\frac{dt}{5} = \frac{1}{5}sin\ t = \frac{1}{2}sin\ (5x)$$

$$\int \frac{1}{\cos^2 x\ tan\ x}dx$$

$$\int \frac{1}{\cos^2 x\ tan\ x}dx = \begin{bmatrix} t = tan\ x \\ dt = \frac{1}{\cos^2 x}dx \end{bmatrix} = \int \frac{1}{t}dt = \ln t = \ln(tan\ x)$$

$$\int sin^4 x\cos x\ dx$$

$$\int sin^4 x\cos x\ dx = \begin{bmatrix} t = sin\ x \\ dt = \cos\ x\ dx \end{bmatrix} = \int t^4 dt = \frac{t^5}{5} = \frac{sin^5 x}{5}$$

$$\int \frac{2^{x+2}}{2^x - 13}dx$$

$$\int \frac{2^{x+2}}{2^x - 13}dx = \begin{bmatrix} t = 2^x - 13 \\ dt = 2^x \ln 2 \, dx \end{bmatrix} = 4\int \frac{dt}{t \cdot \ln 2} = \frac{4}{\ln 2}\ln t = \frac{4}{\ln 2}\ln\left(2^x - 13\right)$$

$$\int \frac{3x^2}{x+7}dx$$

$$\int \frac{3x^2}{x+7}dx = 3\int\left(x - 7 + \frac{49}{x+7}\right)dx = 3\left(\frac{x^2}{2} - 7x + 49\ln(x+7)\right)$$

$$\int x^3 sin(x^4 - \pi\ )dx$$

$$\int x^3 sin(x^4 - \pi\ )dx = \begin{bmatrix} t = x^4 - \pi \\ dt = 4x^3 dx \end{bmatrix} = \frac{1}{4}\int sin\ t\ dt = -\frac{1}{4}\cos t = -\frac{1}{4}\cos\left(x^4 - \pi\ \right)$$

$$\int 5^{tan\,x}(1 + tan^2 x)dx$$

$$\int 5^{tan\,x}(1 + tan^2 x)dx = \begin{bmatrix} t = tan\ x \\ dt = (1 + tan^2 x)dx \end{bmatrix} = \int 5^t\, dt = \frac{5^t}{\ln 5} = \frac{5^{tan\,x}}{\ln 5}$$

$$\int \frac{x^2}{4\sqrt{1 - x^3}}dx$$

$$\int \frac{x^2}{4\sqrt{1 - x^3}}dx = \begin{bmatrix} t = 1 - x^3 \\ dt = -3x^2 dx \end{bmatrix} = -\frac{1}{4}\int \frac{1}{3\sqrt{t}}dt = -\frac{1}{12}\frac{1}{1/2}t^{1/2} = -\frac{\sqrt{t}}{6} = -\frac{\sqrt{1 - x^3}}{6}$$

$$\int \frac{7}{x^2 - 8x + 16}dx$$

$$\int \frac{7}{x^2 - 8x + 16}dx = 7\int \frac{1}{(x-4)^2}dx = \begin{bmatrix} t = x - 4 \\ dt = dx \end{bmatrix} = 7\int t^{-2}dt = -\frac{7}{1}t^{-1} = -\frac{7}{x-4}$$

$$\int \frac{5\cos\sqrt{x}}{\sqrt{x}}dx$$

$$\int \frac{5\cos\sqrt{x}}{\sqrt{x}}dx = \begin{bmatrix} t = \sqrt{x} \\ dt = \dfrac{dx}{2\sqrt{x}} \end{bmatrix} = 5\int 2\cos t\ dt = 10\ sin\ t = 10\ sin\ \sqrt{x}$$

$$\int \frac{(1 + \ln x)^2}{4x}\ dx$$

$$\int \frac{(1 + \ln x)^2}{4x}\ dx = \begin{bmatrix} t = 1 + \ln x \\ dt = \dfrac{1}{x}dx \end{bmatrix} = \int \frac{t^2}{4}dt = \frac{1}{4}\frac{t^3}{3} = \frac{(1 + \ln x)^3}{12}$$

$$\int \frac{1}{x^2} sin\ \frac{1}{x}dx$$

$$\int \frac{1}{x^2} sin\ \frac{1}{x}dx = \begin{bmatrix} t = \dfrac{1}{x} \\ dt = -\dfrac{1}{x^2}dx \end{bmatrix} = -\int sin\ t\ dt = \cos t = \cos\left(\frac{1}{x}\right)$$

$$\int \frac{1}{x^2 + 4x + 5}dx$$

$$\int \frac{1}{x^2+4x+5}dx = \int \frac{1}{(x+2)^2+1}dx = \begin{bmatrix} t = x+2 \\ dt = dx \end{bmatrix} = \int \frac{1}{t^2+1}dt = arc\ tan\ t = arc\ tan\ (x+2)$$

$$\int x\sqrt[3]{x^2+1}dx$$

$$\int x\sqrt[3]{x^2+1}dx = \begin{bmatrix} t = x^2+1 \\ dt = 2xdx \end{bmatrix} = \int \frac{1}{2}\sqrt{t}dt = \frac{1}{2}\frac{3}{4}t^{4/3} = \frac{3}{8}\sqrt[3]{(x^2+1)^4}$$

$$\int x\ sin\ x\ dx$$

$$\int x\ sin\ x\ dx = \begin{bmatrix} u = x & dv = sin\ x\ dx \\ du = dx & v = -\cos\ x \end{bmatrix} = -x\cos\ x - \int -cox\ x\ dx = -x\cos x + sin\ x$$

$$\int e^{2x} sin\ x\ dx$$

$$\int e^{2x} sin\ x\ dx = \begin{bmatrix} u = e^{2x} & dv = sin\ x\ dx \\ du = 2e^{2x}dx & v = -\cos x \end{bmatrix} = -e^{2x}\cos x + 2\int e^{2x}\cos x dx = \begin{bmatrix} u = e^{2x} & dv = \cos x\ dx \\ du = 2e^{2x}dx & v = sinx \end{bmatrix}$$

$$-e^{2x}\cos x + 2\left(e^{2x}sinx - 2\int e^{2x} sin\ x\ dx\right) \Rightarrow I = -e^{2x}\cos x + 2e^{2x} sin\ x - 4I \Rightarrow$$

$$I = \frac{-e^{2x}\cos x + 2e^{2x} sin\ x}{5} = \frac{e^{2x}}{5}(2sin\ x - \cos\ x)$$

$$\int \ln x\ dx$$

$$\int \ln x\ dx = \begin{bmatrix} u = \ln x & dv = dx \\ du = \frac{1}{x}dx & v = x \end{bmatrix} = x\ln x - \int x\frac{1}{x}dx = x\ln x - x = x(\ln x - 1)$$

$$\int x\ln x dx$$

$$\int x\ln x dx = \begin{bmatrix} u = \ln x & dv = xdx \\ du = \frac{1}{x}dx & v = \frac{x^2}{2} \end{bmatrix} = \frac{x^2\ln x}{2} - \int \frac{x^2}{2}\frac{1}{x}dx = \frac{x^2\ln x}{2} - \int \frac{x}{2}dx =$$

$$\frac{x^2\ln x}{2} - \frac{x^2}{4} = \frac{x^2}{2}\left(\ln x - \frac{1}{2}\right)$$

$$\int 2^x xdx$$

$$\int 2^x xdx = \begin{bmatrix} u = x & dv = 2^x dx \\ du = dx & v = \frac{2^x}{\ln 2} \end{bmatrix} = \frac{x2^x}{\ln 2} - \int \frac{2^x dx}{\ln 2} = \frac{x2^x}{\ln 2} - \frac{1}{\ln 2}\frac{2^x}{\ln 2} = \frac{x2^x}{\ln 2} - \frac{2^x}{\ln^2 2}$$

$$\int arc\ sin\ x\ dx$$

$$\int arc\ sin\ x\ dx = \begin{bmatrix} u = arc\ sin\ x & dv = dx \\ du = \frac{1}{\sqrt{1-x^2}}dx & v = x \end{bmatrix} = x\ arc\ sin\ x - \int \frac{xdx}{\sqrt{1-x^2}} = \begin{bmatrix} t = 1-x^2 \\ dt = -2xdx \end{bmatrix} =$$

$$x\ arc\ sin\ x - \int -\frac{1}{2}\frac{dt}{\sqrt{t}} = x\ arc\ sin\ x + \frac{1}{2}\frac{1}{1/2}t^{1/2} = x\ arc\ sin\ x + \sqrt{1-x^2}$$

$$\int (x+2)e^{3x}dx$$

$$\int (x+2)e^{3x}dx = \begin{bmatrix} u = x+2 & dv = e^{3x}dx \\ du = dx & v = \dfrac{e^{3x}}{3} \end{bmatrix} = (x+2)\frac{e^{3x}}{3} - \frac{1}{3}\int e^{3x}dx = (x+2)\frac{e^{3x}}{3} - \frac{1}{3}\frac{1}{3}e^{3x} =$$

$$e^{3x}\left(\frac{x+2}{3} - \frac{1}{9}\right)$$

$$\int \frac{x}{3^x}dx$$

$$\int \frac{x}{3^x}dx = \begin{bmatrix} u = x & dv = 3^{-x}dx \\ du = dx & v = -\dfrac{3^{-x}}{\ln 3} \end{bmatrix} = -\frac{x3^{-x}}{\ln 3} + \int \frac{3^{-x}}{\ln 3}dx = -\frac{x3^{-x}}{\ln 3} - \frac{1}{\ln 3}\frac{3^{-x}}{\ln 3} = -\frac{1}{3^x \ln 3}\left(x + \frac{1}{\ln 3}\right)$$

$$\int (3x-2)\cos x dx$$

$$\int (3x-2)\cos x dx = \begin{bmatrix} u = 3x+2 & dv = \cos x dx \\ du = 3dx & v = \sin x \end{bmatrix} = (3x+2)\sin x - \int 3\sin x\, dx =$$

$$(3x+2)\sin x + 3\cos x$$

$$\int \frac{x}{e^{2x}}dx$$

$$\int \frac{x}{e^{2x}}dx = \begin{bmatrix} u = x & dv = e^{-2x}dx \\ du = dx & v = -\dfrac{e^{-2x}}{2} \end{bmatrix} = -\frac{xe^{-2x}}{2} + \frac{1}{2}\int e^{-2x}dx = -\frac{xe^{-2x}}{2} - \frac{e^{-2x}}{4} =$$

$$-\frac{1}{2e^{2x}}\left(x + \frac{1}{2}\right)$$

$$\int x^2 e^{5x}dx$$

$$\int x^2 e^{5x}dx = \begin{bmatrix} u = x^2 & dv = e^{5x}dx \\ du = 2xdx & v = \dfrac{e^{5x}}{5} \end{bmatrix} = \frac{x^2 e^{5x}}{5} - \frac{2}{5}\int xe^{5x}dx = \begin{bmatrix} u = x & dv = e^{5x}dx \\ du = dx & v = \dfrac{e^{5x}}{5} \end{bmatrix} =$$

$$\frac{x^2 e^{5x}}{5} - \frac{2}{5}\left(\frac{xe^{5x}}{5} - \frac{1}{5}\int e^{5x}dx\right) = \frac{x^2 e^{5x}}{5} - \frac{2xe^{5x}}{25} + \frac{2e^{5x}}{125} = \frac{e^{5x}}{5}\left(x^2 - \frac{2x}{5} + \frac{2}{25}\right)$$

$$\int x^3 \sin x\, dx$$

$$\int x^3 \sin x\, dx = \begin{bmatrix} u = x^3 & dv = \sin x\, dx \\ dx = 3x^2 dx & v = -\cos x \end{bmatrix} = -x^3\cos x + 3\int x^2 \cos x = \begin{bmatrix} u = x^2 & dv = \cos x dx \\ du = 2xdx & v = \sin x \end{bmatrix} =$$

$$-x^3\cos x + 3\left(x^2 \sin x - 2\int x\sin x\, dx\right) = \begin{bmatrix} u = x & dv = \sin x\, dx \\ du = dx & v = -\cos x \end{bmatrix} =$$

$$-x^3\cos x + 3\left(x^2 \sin x - 2\left(-x\cos x + \int \cos x dx\right)\right) = -x^3\cos x + 3\left(x^2 \sin x - 2\left(-x\cos x + \sin x\right)\right) =$$

$$-x^3\cos x + 3x^2 \sin x + 6x\cos x - 6\sin x = \left(-x^3 + 6x\right)\cos x + (3x^2 - 6)\sin x$$

$$\int \left(x^2 + 4\right)3^x dx$$

$$\int\left(x^2+4\right)3^x dx=\begin{bmatrix} u=x^2+4 & dv=3^x dx \\ du=2xdx & v=\dfrac{3^x}{\ln 3}\end{bmatrix}=\frac{\left(x^2+4\right)3^x}{\ln 3}-\frac{2}{\ln 3}\int x3^x dx=$$

$$\begin{bmatrix} u=x & dv=3^x dx \\ du=dx & v=\dfrac{3^x}{\ln 3}\end{bmatrix}=\frac{\left(x^2+4\right)3^x}{\ln 3}-\frac{2}{\ln 3}\left(\frac{x3^x}{\ln 3}-\frac{1}{\ln 3}\int 3^x dx\right)=$$

$$\frac{\left(x^2+4\right)3^x}{\ln 3}-\frac{2}{\ln 3}\left(\frac{x3^x}{\ln 3}-\frac{1}{\ln 3}\left(\frac{3^x}{\ln 3}\right)\right)=\frac{3^x}{\ln 3}\left(x^2+4-\frac{2x}{\ln 3}+\frac{2}{\ln^2 3}\right)$$

$$\int x^2\cos x dx$$

$$\int x^2\cos x dx=\begin{bmatrix} u=x^2 & dv=\cos x dx \\ du=2xdx & v=sin\ x\end{bmatrix}=x^2 sin\ x-2\int x sin\ x\ dx=\begin{bmatrix} u=x & dv=sin\ x\ dx \\ du=dx & v=-\cos x\end{bmatrix}=$$

$$x^2 sin\ x-2\left(-x\cos x+\int\cos x dx\right)=x^2 sin\ x-2\left(-x\cos x+sin\ x\right)=(x^2-2)sin\ x+2x\cos x$$

$$\int\frac{x^2}{2^x}dx$$

$$\int\frac{x^2}{2^x}dx=\begin{bmatrix} u=x^2 & dv=2^{-x}dx \\ du=2xdx & v=-\dfrac{2^{-x}}{\ln 2}\end{bmatrix}=-\frac{x^2 2^{-x}}{\ln 2}-\frac{2}{\ln 2}\int x2^{-x}dx=\begin{bmatrix} u=x & dv=2^{-x}dx \\ du=dx & v=-\dfrac{2^{-x}}{\ln 2}\end{bmatrix}=$$

$$-\frac{x^2 2^{-x}}{\ln 2}-\frac{2}{\ln 2}\left(-\frac{x2^{-x}}{\ln 2}+\frac{1}{\ln 2}\int 2^{-x}dx\right)=-\frac{x^2 2^{-x}}{\ln 2}-\frac{2}{\ln 2}\left(-\frac{x2^{-x}}{\ln 2}+\frac{1}{\ln 2}\left(-\frac{2^{-x}}{\ln 2}\right)\right)=$$

$$\frac{1}{2^x\ln 2}\left(-x^2+\frac{2x}{\ln 2}+\frac{2}{\ln^2 2}\right)$$

$$\int(1-x^2)2^{3x}dx$$

$$\int(1-x^2)2^{3x}dx=\begin{bmatrix} u=1-x^2 & dv=2^{3x}dx \\ du=-2xdx & v=\dfrac{2^{3x}}{3\ln 2}\end{bmatrix}=(1-x^2)\frac{2^{3x}}{3\ln 2}+\frac{2}{3\ln 2}\int x2^{3x}dx=$$

$$\begin{bmatrix} u=x & dv=2^{3x}dx \\ du=dx & v=\dfrac{2^{3x}}{3\ln 2}\end{bmatrix}=(1-x^2)\frac{2^{3x}}{3\ln 2}+\frac{2}{3\ln 2}\left(x\frac{2^{3x}}{3\ln 2}-\int\frac{2^{3x}}{3\ln 2}dx\right)=$$

$$(1-x^2)\frac{2^{3x}}{3\ln 2}+\frac{2}{3\ln 2}\left(x\frac{2^{3x}}{3\ln 2}-\frac{1}{3\ln 2}\frac{2^{3x}}{3\ln 2}\right)=\frac{2^{3x}}{3\ln 2}\left(1-x^2+\frac{2x}{3\ln 2}-\frac{2}{3^2\ln^2 2}\right)$$

$$\int x^2 sin3xdx$$

$$\int x^2 sin3xdx=\begin{bmatrix} u=x^2 & dv=sin3xdx \\ du=2xdx & v=-\dfrac{\cos 3x}{3}\end{bmatrix}=-\frac{x^2\cos 3x}{3}+\frac{2}{3}\int x\cos 3xdx=\begin{bmatrix} u=x & dv=\cos 3xdx \\ du=dx & v=\dfrac{sin3x}{3}\end{bmatrix}=$$

$$-\frac{x^2\cos 3x}{3}+\frac{2}{3}\left(\frac{xsin3x}{3}-\frac{1}{3}\int sin3xdx\right)=-\frac{x^2\cos 3x}{3}+\frac{2}{3}\left(\frac{xsin3x}{3}-\frac{1}{3}\left(\frac{-\cos 3x}{3}\right)\right)=$$

$$\frac{-x^2\cos 3x}{3}+\frac{2xsin3x}{9}+\frac{2\cos 3x}{27}$$

$$\int\cos(\ln x)dx$$

$$\int \cos(\ln x)dx = \begin{bmatrix} u = \cos(\ln x) & dv = dx \\ du = -sin(\ln x)\cdot\frac{1}{x} & v = x \end{bmatrix} = x\cos(\ln x) + \int sin(\ln x)dx = \begin{bmatrix} u = sin(\ln x) & dv = dx \\ du = \cos(\ln x)\cdot\frac{1}{x} & v = x \end{bmatrix} =$$

$$x\cos(\ln x) + xsin(\ln x) - \int \cos(\ln x)dx \quad \Rightarrow \quad I = x\cos(\ln x) + xsin(\ln x) - I \quad \Rightarrow$$

$$I = \frac{x\cos(\ln x) + xsin(\ln x)}{2}$$

$$\int x^3 \ln x dx$$

$$\int x^3 \ln x dx = \begin{bmatrix} u = \ln x & dv = x^3 dx \\ du = \frac{1}{x}dx & v = \frac{x^4}{4} \end{bmatrix} = \frac{x^4 \ln x}{4} - \int \frac{x^4}{4x}dx = \frac{x^4 \ln x}{4} - \frac{x^4}{16}$$

$$\int e^{4x}\cos 4x dx$$

$$\int e^{4x}\cos 4x dx = \begin{bmatrix} u = e^{4x} & dv = \cos 4x dx \\ du = 4e^{4x}dx & v = \frac{sin 4x}{4} \end{bmatrix} = \frac{e^{4x} sin 4x}{4} - \int e^{4x} sin 4x dx =$$

$$\begin{bmatrix} u = e^{4x} & dv = sin 4x dx \\ du = 4e^{4x}dx & v = \frac{-\cos 4x}{4} \end{bmatrix} = \frac{e^{4x} sin 4x}{4} + \left(\frac{e^{4x}\cos 4x}{4} + \int e^{4x}\cos 4x dx\right) \quad \Rightarrow$$

$$I = \frac{e^{4x} sin 4x}{4} + \frac{e^{4x}\cos 4x}{4} + I \quad \Rightarrow \quad I = \frac{e^{4x}(sin 4x + \cos 4x)}{8}$$

$$\int (x-1)5^x dx$$

$$\int (x-1)5^x dx = \begin{bmatrix} u = x-1 & dv = 5^x dx \\ du = dx & v = \frac{5^x}{\ln 5} \end{bmatrix} = (x-1)\frac{5^x}{\ln 5} - \frac{1}{\ln 5}\int 5^x dx = (x-1)\frac{5^x}{\ln 5} - \frac{1}{\ln 5}\cdot\frac{5^5}{\ln 5} =$$

$$\frac{5^x}{\ln 5}\left(x - 1 - \frac{1}{\ln 5}\right)$$

$$\int \ln^2 x dx$$

$$\int \ln^2 x dx = \begin{bmatrix} u = \ln^2 x & dv = dx \\ du = 2\ln x\frac{1}{x} & v = x \end{bmatrix} = x\ln^2 x - 2\int \ln x dx = \begin{bmatrix} u = \ln x & dv = dx \\ du = \frac{1}{x}dx & v = x \end{bmatrix} =$$

$$x\ln^2 x - 2\left(x\ln x - \int dx\right) = x\ln^2 x - 2x\ln x - 2x$$

$$\int sin^2 x dx$$

$$\int sin^2 x dx = \begin{bmatrix} u = sinx & dv = sinxdx \\ du = \cos x dx & v = -\cos x \end{bmatrix} = -sinx\cos x + \int \cos^2 x dx = -sinx\cos x + \int \left(1 - sin^2 x\right)dx =$$

$$-sinx\cos x + \int dx - \int sin^2 x dx \quad \Rightarrow \quad I = x - sinx\cos x - I \quad \Rightarrow \quad I = \frac{x - sinx\cos x}{2}$$

$$\int sin^2 x dx$$

$$\int sin^2 xdx = \begin{bmatrix} \cos 2x = \cos^2 x - sin^2 x & 1-\cos 2x = 2sin^2 x \\ \cos 2x = 1-2sin^2 x & \dfrac{1-\cos 2x}{2} = sin^2 x \end{bmatrix} = \int \left(\frac{1-\cos 2x}{2} \right) dx =$$

$$\int \frac{1}{2} dx - \frac{1}{2} \int \cos 2x dx = \begin{bmatrix} t = 2x \\ dt = 2dx \end{bmatrix} = \frac{1}{2} x - \frac{1}{2} \frac{sin2x}{2} = \frac{x - sinx \cos x}{2}$$

$$\int \cos^2 xdx$$

$$\int \cos^2 xdx = \begin{bmatrix} u = \cos x & dv = \cos xdx \\ du = -sinxdx & v = sinx \end{bmatrix} = sinx \cos x + \int sin^2 xdx = sinx \cos x + \int (1 - \cos^2 x) dx =$$

$$sinx \cos x + \int dx - \int \cos^2 xdx \quad \Rightarrow \quad I = sinx \cos x + x - I \quad \Rightarrow I = \frac{x + sinx \cos x}{2}$$

$$\int \frac{\ln x}{x^2} dx$$

$$\int \frac{\ln x}{x^2} dx = \begin{bmatrix} u = \ln x & dv = x^{-2} dx \\ du = \dfrac{1}{x} dx & v = -x^{-1} \end{bmatrix} = -\frac{\ln x}{x} + \int \frac{1}{x^2} dx = -\frac{\ln x}{x} - \frac{1}{x} = -\frac{1 + \ln x}{x}$$

$$\int tan^2 xdx$$

$$\int tan^2 xdx = \int (tan^2 x + 1 - 1) dx = \int (tan^2 x + 1) dx - \int 1dx = tan\ x - x$$

$$\int tan^2 xdx$$

$$\int tan^2 xdx = \begin{bmatrix} t = tanx \\ dt = (1 + tan^2 x)dx \end{bmatrix} \Rightarrow dx = \frac{dt}{1 + tan^2 x} = \frac{dt}{1 + t^2} \Bigg] = \int t^2 \frac{dt}{1+t^2} = \int \frac{t^2}{1+t^2} dt =$$

$$\int \left(1 - \frac{1}{1+t^2} \right) dt = \int dt - \int \frac{1}{1+t^2} dt = t - arc\ tan\ t = tanx - arc\ tan(tanx) = tanx - x$$

$$\int \frac{x}{\cos^2 x} dx$$

$$\int \frac{x}{\cos^2 x} dx = \begin{bmatrix} u = x & dv = \dfrac{1}{\cos^2 x} dx \\ du = dx & v = tanx \end{bmatrix} = xtanx - \int tanxdx = xtanx - \int \frac{sinx}{\cos x} dx =$$

$$xtanx + \ln(\cos x)$$

$$\int e^x sin\ x\ dx$$

$$\int e^x sin\ x\ dx = \begin{bmatrix} u = e^x & dv = sinxdx \\ du = e^x dx & v = -\cos x \end{bmatrix} = -e^x \cos x + \int e^x \cos xdx = \begin{bmatrix} u = e^x & dv = \cos xdx \\ du = e^x dx & v = sin\ x \end{bmatrix} =$$

$$-e^x \cos x + e^x sinx - \int e^x sinxdx \quad \Rightarrow \quad I = -e^x \cos x + e^x sinx - I \quad \Rightarrow$$

$$I = \frac{e^x (sinx - \cos x)}{2}$$

$$\int \frac{10}{x^2 - 4x + 4} dx$$

$$\int \frac{10}{x^2-4x+4}dx = \int \frac{10}{(x-2)^2 dx} = 10\int (x-2)^{-2}dx = -\frac{10}{x-2}$$

$$\int \frac{3x+2}{x^2-9}dx$$

$$\int \frac{3x+2}{x^2-9}dx = \left[\frac{3x+2}{x^2-9} = \frac{A}{x+3} + \frac{B}{x-3}\right] = \begin{bmatrix} A = 11/6 \\ B = 7/6 \end{bmatrix} = \int \frac{11/6}{x+3}dx + \int \frac{7/6}{x-3}dx =$$

$$\frac{11}{6}\ln(x+3) + \frac{7}{6}\ln(x-3)$$

$$\int \frac{x^2-3}{x(x-1)(x+2)}dx$$

$$\int \frac{x^2-3}{x(x-1)(x+2)}dx = \left[\frac{x^2-3}{x(x-1)(x+2)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+2}\right] = \begin{bmatrix} A = 3/2 \\ B = -2/3 \\ C = 1/6 \end{bmatrix} =$$

$$\int \frac{3/2}{x}dx - \int \frac{2/3}{x-1}dx + \int \frac{1/6}{x+2}dx = \frac{3}{2}\ln x - \frac{2}{3}\ln(x-1) + \frac{1}{6}\ln(x+2)$$

$$\int \frac{3}{x^2-x}dx$$

$$\int \frac{3}{x^2-x}dx = \left[\frac{3}{x^2-x} = \frac{A}{x} + \frac{B}{x-1}\right] = \begin{bmatrix} A = -3 \\ B = 3 \end{bmatrix} = \int \frac{-3}{x}dx + \int \frac{3}{x-1}dx = -3\ln x + 3\ln(x-1)$$

$$\int \frac{1-2x}{x^3-6x^2+11x-6}dx$$

$$\int \frac{1-2x}{x^3-6x^2+11x-6}dx = \left[\frac{1-2x}{x^3-6x^2+11x-6} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}\right] = \begin{bmatrix} A = -1/2 \\ B = 3 \\ C = -5/2 \end{bmatrix} =$$

$$\int \frac{-1/2}{x-1}dx + \int \frac{3}{x-2}dx + \int \frac{-5/2}{x-3}dx = -\frac{1}{2}\ln(x-1) + 3\ln(x-2) - \frac{5}{2}\ln(x-3)$$

$$\int \frac{x-3}{x^2-6x+5}dx$$

$$\int \frac{x-3}{x^2-6x+5}dx = \begin{bmatrix} t = x^2-6x+5 \\ dt = (2x-6)dx = 2(x-3)dx \end{bmatrix} = \int \frac{1/2}{t}dt = \frac{1}{2}\ln t = \frac{1}{2}\ln(x^2-6x+5)$$

$$\int \frac{x^2}{x-5}dx$$

$$\int \frac{x^2}{x-5}dx = \left[\frac{x^2}{x-5} = x+5+\frac{25}{x-5}\right] = \int xdx + \int 5dx + \int \frac{25}{x-5}dx = \frac{x^2}{x} + 5x + 25\ln(x-5)$$

$$\int \frac{x^3-4}{x^2-2x}dx$$

$$\int \frac{x^3-4}{x^2-2x}dx = \left[\frac{x^3-4}{x^2-2x} = x+2+\frac{4x-4}{x^2-2x}\right] = \int xdx + \int 2dx + \int \frac{4x-4}{x^2-2x}dx = \begin{bmatrix} t = x^2-2x \\ dt = (2x-2)dx \end{bmatrix} =$$

$$\frac{x^2}{2} + 2x + 2\ln(x^2-2x)$$

$$\int \frac{1}{x^2(x-1)^2}dx$$

$$\int \frac{1}{x^2(x-1)^2}dx = \left[\frac{1}{x^2(x-1)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} + \frac{D}{(x-1)^2}\right] \Rightarrow$$

$$Ax(x-1)^2 + B(x-1)^2 + Cx^2(x-1) + Dx^2 = 1 \quad \Rightarrow \begin{cases} A = 2 & B = 1 \\ C = -2 & D = 1 \end{cases} \quad \Rightarrow$$

$$\int \frac{2}{x}dx + \int \frac{1}{x^2}dx + \int \frac{-2}{x-1}dx + \int \frac{1}{(x-1)^2}dx = 2\ln x - \frac{1}{x} - 2\ln(x-1) - \frac{1}{x-1}$$

$$\int \frac{x^2+9}{x^2-9}dx$$

$$\int \frac{x^2+9}{x^2-9}dx = \left[\frac{x^2+9}{x^2-9} = 1 + \frac{18}{x^2-9} = 1 + \frac{A}{x-3} + \frac{B}{x+3}\right] \quad \left[\begin{matrix} A = 3 \\ B = -3 \end{matrix}\right]$$

$$\int 1dx + \int \frac{3}{x-3}dx + \int \frac{-3}{x+3}dx = x + 3\ln(x-3) - 3\ln(x+3)$$

$$\int \frac{2x-1}{x^2-5x+6}dx$$

$$\int \frac{2x-1}{x^2-5x+6}dx = \left[\frac{2x-1}{x^2-5x+6} = \frac{A}{x-2} + \frac{B}{x-3}\right] \Rightarrow \left[\begin{matrix} A = -3 \\ B = 5 \end{matrix}\right] = \int \left(\frac{-3}{x-2} + \frac{5}{x-3}\right)dx =$$

$$-3\ln(x-2) + 5\ln(x-3)$$

$$\int \frac{x-5}{x^2-x-2}dx$$

$$\int \frac{x-5}{x^2-x-2}dx = \left[\frac{x-5}{x^2-x-2} = \frac{A}{x+1} + \frac{B}{x-2}\right] \Rightarrow \left[\begin{matrix} A = 2 \\ B = -1 \end{matrix}\right] = \int \left(\frac{2}{x+1} + \frac{-1}{x-2}\right)dx =$$

$$2\ln(x+1) - \ln(x-2)$$

$$\int \frac{x^4}{x^2-1}dx$$

$$\int \frac{x^4}{x^2-1}dx = \int \left(x^2 + 1 + \frac{1}{x^2-1}\right)dx = \left[\frac{1}{x^2-1} = \frac{A}{x+1} + \frac{B}{x-1}\right] \Rightarrow \left[\begin{matrix} A = -1/2 \\ b = 1/2 \end{matrix}\right] =$$

$$\int \left(x^2 + 1 + \frac{-1/2}{x-1} + \frac{1/2}{x+1}\right)dx = \frac{x^3}{3} + x - \frac{1}{2}\ln(x-1) + \frac{1}{2}\ln(x-1)$$

$$\int \frac{x+1}{x^2-5x+6}dx$$

$$\int \frac{x+1}{x^2-5x+6}dx = \left[\frac{x+1}{x^2-5x+6} = \frac{A}{x-2} + \frac{B}{x-3}\right] \Rightarrow \left[\begin{matrix} A = -3 \\ B = 4 \end{matrix}\right] = \int \left(\frac{-3}{x-2} + \frac{4}{x-3}\right)dx =$$

$$-3\ln(x-2) + 4\ln(x-3)$$

$$\int \frac{4x^2-15x+13}{x^3-6x^2+11x-6}dx$$

$$\int \frac{4x^2-15x+13}{x^3-6x^2+11x-6}\,dx = \left[\frac{4x^2-15x+13}{x^3-6x^2+11x-6} = \frac{A}{x-2}+\frac{B}{x-3}+\frac{C}{x-1}\right] \Rightarrow \begin{bmatrix} A=1 \\ B=2 \\ C=1 \end{bmatrix} =$$

$$\int \left(\frac{1}{x-2}+\frac{2}{x-3}+\frac{1}{x-1}\right)dx = \ln(x-2)+2\ln(x-3)+\ln(x-1)$$

$$\int \frac{x^3}{x^2-1}\,dx$$

$$\int \frac{x^3}{x^2-1}\,dx = \int\left(x+\frac{x}{x^2-1}\right)dx = \left[\frac{x}{x^2-1} = \frac{A}{x+1}+\frac{B}{x-1}\right] \Rightarrow \begin{bmatrix} A=1/2 \\ B=1/2 \end{bmatrix} =$$

$$\int\left(x+\frac{1/2}{x+1}+\frac{1/2}{x-1}\right)dx = \frac{x^2}{x}+\frac{1}{2}\ln(x+1)+\frac{1}{2}\ln(x-1)$$

$$\int \frac{3x+3}{x^2+x-2}\,dx$$

$$\int \frac{3x+3}{x^2+x-2}\,dx = \left[\frac{3x+3}{x^2+x-2} = \frac{A}{x-1}+\frac{B}{x+2}\right] \Rightarrow \begin{bmatrix} A=2 \\ B=1 \end{bmatrix} = \int\left(\frac{2}{x-1}+\frac{1}{x+2}\right)dx =$$

$$2\ln(x-1)+\ln(x+2)$$

$$\int \frac{x+1}{x^2-3x+2}\,dx$$

$$\int \frac{x+1}{x^2-3x+2}\,dx = \left[\frac{x+1}{x^2-3x+2} = \frac{A}{x-1}+\frac{B}{x-2}\right] \Rightarrow \begin{bmatrix} A=-2 \\ B=3 \end{bmatrix} = \int\left(\frac{-2}{x-1}+\frac{3}{x-2}\right)dx =$$

$$-2\ln(x-1)+3\ln(x-2)$$

$$\int \frac{3x^2+2x-7}{x^3-2x^2-x+2}\,dx$$

$$\int \frac{3x^2+2x-7}{x^3-2x^2-x+2}\,dx = \left[\frac{3x^2+2x-7}{x^3-2x^2-x+2} = \frac{A}{x-1}+\frac{B}{x+1}+\frac{C}{x-2}\right] \Rightarrow \begin{bmatrix} A=1 \\ B=-1 \\ C=3 \end{bmatrix} =$$

$$\int\left(\frac{1}{x-1}+\frac{-1}{x+1}+\frac{3}{x-2}\right)dx = \ln(x-1)-\ln(x+1)+3\ln(x-2)$$

$$\int \frac{5x^3+18x^2+14x-24}{x^4+3x^3-10x^2-24x}\,dx$$

$$\int \frac{5x^3+18x^2+14x-24}{x^4+3x^3-10x^2-24x}\,dx = \left[\frac{5x^3+18x^2+14x-24}{x^4+3x^3-10x^2-24x} = \frac{A}{x}+\frac{B}{x+2}+\frac{C}{x-3}+\frac{D}{x+4}\right] \Rightarrow$$

$$\begin{bmatrix} A=1 & C=3 \\ B=-1 & D=2 \end{bmatrix} = \int\left(\frac{1}{x}+\frac{-1}{x+2}+\frac{3}{x-3}+\frac{2}{x+4}\right)dx =$$

$$\ln x-\ln(x+2)+3\ln(x-3)+2\ln(x+4)$$

$$\int \frac{dx}{x^2+9}$$

$$\int \frac{dx}{x^2+9} = \frac{1}{9}\int \frac{dx}{\frac{x^2}{9}+1} = \begin{bmatrix} t=\dfrac{x}{3} \\ dt=\dfrac{dx}{3} \end{bmatrix} = \frac{1}{9}\int\frac{3dt}{t^2+1} = \frac{1}{3}arc\ tan\ t = \frac{1}{3}arc\ tan\left(\frac{x}{3}\right)$$

$$\int \frac{3dx}{x^2+2}$$

$$\int \frac{3dx}{x^2+2} = \frac{3}{2}\int \frac{dx}{\frac{x^2}{2}+1} = \begin{bmatrix} t = \frac{x}{\sqrt{2}} \\ dt = \frac{dx}{\sqrt{2}} \end{bmatrix} = \frac{3}{2}\int \frac{\sqrt{2}}{t^2+1}\, dt = \frac{3\sqrt{2}}{2} arc\ tan\ t = \frac{3\sqrt{2}}{2} arc\ tan\left(\frac{x}{\sqrt{2}}\right)$$

$$\int \frac{dx}{2x^2+1}$$

$$\int \frac{dx}{2x^2+1} = \begin{bmatrix} t = \sqrt{2}x \\ dt = \sqrt{2}dx \end{bmatrix} = \int \frac{dt/\sqrt{2}}{t^2+1} = \frac{1}{\sqrt{2}} arc\ tan\ t = \frac{1}{\sqrt{2}} arc\ tan\left(\sqrt{2}x\right)$$

$$\int \frac{3}{3x^2+\frac{1}{2}} dx$$

$$\int \frac{3}{3x^2+\frac{1}{2}} dx = 3\int \frac{dx}{3x^2+\frac{1}{2}} = 3\int \frac{2dx}{6x^2+1} = \begin{bmatrix} t = \sqrt{6}x \\ dt = \sqrt{6}dx \end{bmatrix} = 6\int \frac{dt/\sqrt{6}}{t^2+1} = \frac{6}{\sqrt{6}} arc\ tan\ t = \frac{6}{\sqrt{6}} arc\ tan\left(\sqrt{6}x\right)$$

$$\int \frac{dx}{x^2-8x+18}$$

$$\int \frac{dx}{x^2-8x+18} = \int \frac{dx}{(x-4)^2+2} = \begin{bmatrix} t = x-4 \\ dt = dx \end{bmatrix} = \int \frac{dt}{t^2+2} = \int \frac{dt/2}{t^2/2+1} = \begin{bmatrix} u = t/\sqrt{2} \\ du = dt/\sqrt{2} \end{bmatrix} =$$

$$\frac{1}{2}\int \frac{\sqrt{2}du}{u^2+1} = \frac{\sqrt{2}}{2} arc\ tan\ u = \frac{\sqrt{2}}{2} arc\ tan\ \frac{t}{\sqrt{2}} = \frac{\sqrt{2}}{2} arc\ tan\ \frac{x-4}{\sqrt{2}}$$

$$\int \frac{dx}{x+\sqrt{x}}$$

$$\int \frac{dx}{x+\sqrt{x}} = \begin{bmatrix} t^2 = x \\ 2tdt = dx \end{bmatrix} = \int \frac{2tdt}{t^2+t} = 2\int \frac{tdt}{t(t+1)} = 2\int \frac{dt}{t+1} = 2\ln(t+1) = 2\ln(\sqrt{x}+1)$$

$$\int \frac{1-e^x}{e^{2x}} dx$$

$$\int \frac{1-e^x}{e^{2x}} dx = \begin{bmatrix} t = e^x \Rightarrow x = \ln t \\ dx = \frac{1}{t} dt \end{bmatrix} = \int \frac{1-t}{t^2}\cdot\frac{1}{t} dt = \int \frac{1-t}{t^3} dt = \int t^{-3} dt - \int t^{-2} dt =$$

$$-\frac{1}{2}t^{-2} + t^{-1} = \frac{1}{t} - \frac{1}{2t^2} = \frac{1}{e^x} - \frac{1}{2e^{2x}}$$

$$\int \sqrt{1-x^2}\, dx$$

$$\int \sqrt{1-x^2}\, dx = \begin{bmatrix} sin^2 t = 1-x^2 \\ x = \cos t \\ dx = -sin\ t\ dt \end{bmatrix} = \int sin\ t\ (-sin\ t)\ dt = -\int sin^2 t\ dt = -\frac{t - sin\ t\cos t}{2} =$$

$$-\frac{1}{2}\left(arc\cos x - x\sqrt{1-x^2}\right)$$

$$\int \frac{dx}{sin2x}$$

$$\int \frac{dx}{sin2x} = \int \frac{dx}{2sinx\cos x} = \int \frac{\frac{dx}{\cos x}}{\frac{2sinx\cos x}{\cos x}} = \frac{1}{2}\int \frac{\frac{dx}{\cos^2 x}}{tanx} = \begin{bmatrix} t = tanx \\ dt = \frac{1}{\cos^2 x}dx \end{bmatrix} = \frac{1}{2}\int \frac{dt}{t} =$$

$$\frac{1}{2}\ln t = \frac{1}{2}\ln(tanx)$$

$$\int \frac{dx}{sin2x}$$

$$\int \frac{dx}{sin2x} = \int \frac{dx}{2sinx\cos x} = \frac{1}{2}\int \frac{dx}{sinx\cos x} = \frac{1}{2}\int \frac{sin^2 x + \cos^2 x}{sinx\cos x}dx =$$

$$\frac{1}{2}\left(\int \frac{sin^2 x}{sinx\cos x}dx + \int \frac{\cos^2 x}{sinx\cos x}dx\right) = \frac{1}{2}\left(\int \frac{sinx}{\cos x}dx + \int \frac{\cos x}{sinx}dx\right) =$$

$$\begin{bmatrix} t = \cos x & u = sinx \\ dt = -sinxdx & du = \cos xdx \end{bmatrix} = \frac{1}{2}\left(\int \frac{-dt}{t} + \int \frac{du}{u}\right) = \frac{1}{2}(-\ln t + \ln u) =$$

$$\frac{1}{2}(\ln(sinx) - \ln(\cos x)) = \frac{1}{2}\ln(tanx)$$